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1 Introduction

A general three-dimensional initial-value perturbation problem is presented to study the linear stability of a two-dimensional growing wake. The base flow has been obtained by approximating it with an expansion solution for the longitudinal velocity component that considers the lateral entrainment process [1]. By imposing arbitrary three-dimensional perturbations in terms of the vorticity, the temporal behaviour, including both the early time transient as well as the long time asymptotics, is considered [2], [3], [4]. The approach has been to first perform a Laplace-Fourier transform of the governing viscous disturbance equations and then resolve them numerically by the method of lines. The base model is combined with a change of coordinate [5]. Base flow configurations corresponding to a R of 35, 50, 100 and various physical inputs are examined. In the case of longitudinal disturbances, a comparison with recent spatio-temporal multiscale Orr-Sommerfeld analysis [6], [7] is presented.

2 The initial-value problem

The base flow is viscous and incompressible. To define it, the longitudinal component of an approximated Navier-Stokes expansion for the two-dimensional steady bluff body wake [1], [8] has been used. The x coordinate is parallel to the free stream velocity, the y coordinate is normal. The coordinate x_0 plays the role of parameter of the system together with the Reynolds number. The analytical expression for the wake profile is $U(y; x_0, R) = 1 - a(R)x_0^{-1/2}\mathrm{e}^{-(\mathrm{Ry}^2)/(4\mathrm{x}_0)}$, where $a(R)$ depends on the Reynolds number [8]. By changing x_0, the base flow profile locally approximates the behaviour of the actual wake generated by the body. The equations are

$$\nabla^2\widetilde{v} = \widetilde{\Gamma} \tag{1}$$

$$\frac{\partial\widetilde{\Gamma}}{\partial t} + U\frac{\partial\widetilde{\Gamma}}{\partial x} - \frac{\partial\widetilde{v}}{\partial x}\frac{d^2U}{dy^2} = \frac{1}{R}\nabla^2\widetilde{\Gamma} \tag{2}$$

$$\frac{\partial\widetilde{\omega}_y}{\partial t} + U\frac{\partial\widetilde{\omega}_y}{\partial x} + \frac{\partial\widetilde{v}}{\partial z}\frac{dU}{dy} = \frac{1}{R}\nabla^2\widetilde{\omega}_y \tag{3}$$

where $\widetilde{\omega}_y$ is the transversal component of the perturbation vorticity, while $\widetilde{\Gamma}$ is defined as $\widetilde{\Gamma} = \frac{\partial \widetilde{\omega}_z}{\partial x} - \frac{\partial \widetilde{\omega}_x}{\partial z}$. All physical quantities are normalized with respect to the free stream velocity, the spatial scale of the flow D and the density. By introducing the moving coordinate transform $\xi = x - U_0 t$ [5] and performing a combined Laplace-Fourier decomposition of the dependent variables in terms of ξ and z, the governing equations become

$$\frac{\partial^2 \hat{v}}{\partial y^2} - (k^2 - \alpha_i^2 + 2i\alpha_r\alpha_i)\hat{v} = \hat{\Gamma} \tag{4}$$

$$\begin{aligned}\frac{\partial \hat{\Gamma}}{\partial t} &= -ikcos(\phi)(U - U_0)\hat{\Gamma} + ikcos(\phi)\frac{d^2U}{dy^2}\hat{v} \\ &+ \alpha_i(U - U_0)\hat{\Gamma} - \alpha_i\frac{d^2U}{dy^2}\hat{v} + \frac{1}{R}[\frac{\partial^2\hat{\Gamma}}{\partial y^2} - (k^2 - \alpha_i^2 + 2i\alpha_r\alpha_i)\hat{\Gamma}]\end{aligned} \tag{5}$$

$$\begin{aligned}\frac{\partial \hat{\omega}_y}{\partial t} &= -ikcos(\phi)(U - U_0)\hat{\omega}_y - iksin(\phi)\frac{dU}{dy}\hat{v} \\ &+ \alpha_i(U - U_0)\hat{\omega}_y + \frac{1}{R}[\frac{\partial^2\hat{\omega}_y}{\partial y^2} - (k^2 - \alpha_i^2 + 2i\alpha_r\alpha_i)\hat{\omega}_y]\end{aligned} \tag{6}$$

where $\hat{f}(y, t; \alpha, \gamma) = \int_{-\infty}^{+\infty}\int_{0}^{+\infty} \widetilde{f}(\xi, y, z, t)e^{i\alpha\xi + i\gamma z}d\xi dz$ is the Laplace-Fourier transform of a general dependent variable, $\phi = tan^{-1}(\gamma/\alpha_r)$ is the perturbation angle of obliquity, $k = \sqrt{\alpha_r^2 + \gamma^2}$ is the polar wavenumber and $\alpha_r = kcos(\phi)$, $\gamma = ksin(\phi)$ are the wavenumbers in ξ and z directions respectively. We choose periodic and bounded initial conditions:
CASE I (symmetric initial condition): $\hat{v}(0, y) = e^{-y^2}cos(\beta y)$, $\hat{\omega}_y(0, y) = 0$
CASE II (asymmetric initial condition): $\hat{v}(0, y) = e^{-y^2}sin(\beta y)$, $\hat{\omega}_y(0, y) = 0$

3 Results and Conclusions

The amplification factor G is defined as the normalized energy density [3], namely $G(t; k, \phi) = E(t; k, \phi)/E(t = 0; k, \phi)$. It effectively measures the growth of the energy at time t, for a given initial condition at $t = 0$ (fig. 1). By defining the temporal growth rate [4] as $r = log|E(t)|/(2t)$ ($E(t)$ is the total perturbation energy) and the angular frequency f as the temporal derivative of disturbance phase, we can evaluate the initial stages of exponential growth and, in the case of 2D disturbances, compare them with the normal mode theory results [6] (fig. 2).
Figure 1 yields three differing examples of early transient periods. Case (a) shows that a growing wave becomes damped, increasing the obliquity angle beyond $\pi/4$. Case (b) corresponds to dispersion relation values far from the saddle point and shows that spatially damped/amplified waves can be temporally amplified/damped. Case (c) demonstrates that perturbations normal to the base flow are stable. Figure 2 presents the comparison between the initial value problem and the Orr-Sommerfeld problem. The results are parameterized with respect to the position x_0 through the polar wavenumber $k = k(x_0)$. Equations are integrated in time beyond the transient until the

temporal growth rate asymptotes to a constant value. We observed a very good agreement with the stability characteristics given by the Orr-Sommerfeld theory for both the symmetric and asymmetric arbitrary disturbances considered.

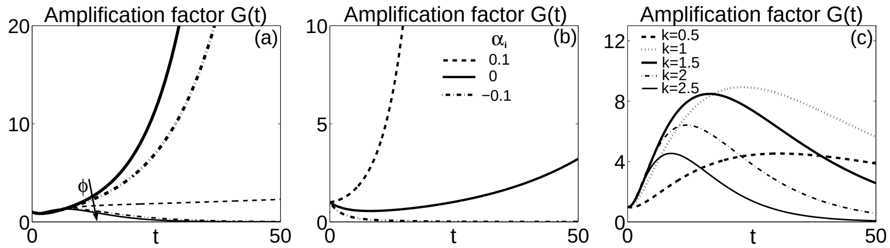

Fig. 1. The amplification factor G as a function of time. **(a)**: $R = 100$, $k = 1.2$, $\alpha_i = -0.1$, $\beta = 1$, $x_0 = 10.15$, $\phi = 0, \pi/8, \pi/4, (3/8)\pi, \pi/2$, symmetric perturbation (case I). **(b)**: $R = 50$, $k = 0.3$, $\beta = 1$, $\phi = 0$, $x_0 = 5.20$, $\alpha_i = -0.1, 0, 0.1$, symmetric perturbation (case I). **(c)**: $R = 100$, $\alpha_i = -0.01$, $\beta = 1$, $\phi = \pi/2$, $x_0 = 7.40$, $k = 0.5, 1, 1.5, 2, 2.5$, symmetric perturbation (case I).

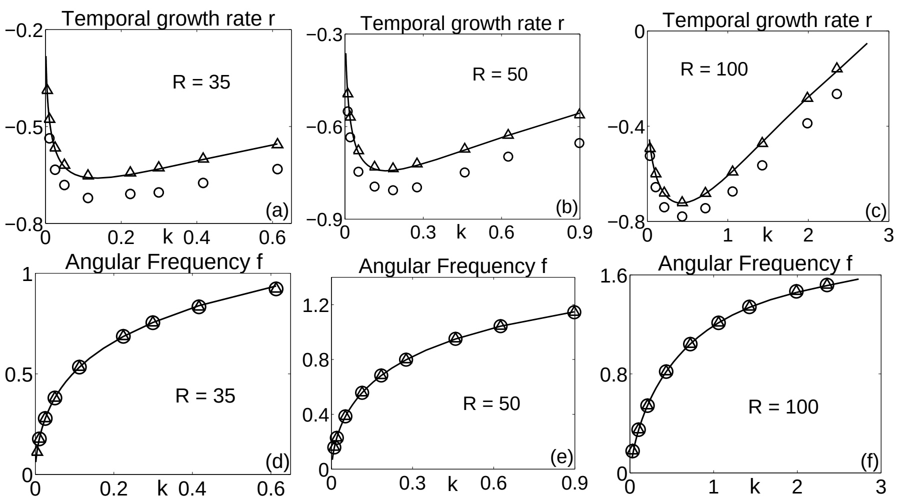

Fig. 2. $\beta = 1$, $\phi = 0$. **(a, b, c)** Temporal growth rate and **(d, e, f)** angular frequency. Comparison between present results (triangles: symmetric perturbation, case I; circles: asymmetric perturbation, case II) and normal mode analysis by Tordella, Scarsoglio and Belan, 2006 Phys. Fluids (solid lines). The wavenumber $\alpha = \alpha_r(x_0) + i\alpha_i(x_0)$, $\alpha_r(x_0) = k(x_0)$ is the most unstable wavenumber in any section of the near-parallel wake (dominant saddle point in the local dispersion relation). The wake sections considered are in the interval $3D \leq x_o \leq 50D$.

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